Preface

Linear algebra and matrix theory, abbreviated here as LAMT, is a foundation for many advanced topics in mathematics and an essential tool for computer science, physics, engineering, bioinformatics, economics, and social sciences. A first course in linear algebra for engineers is like a cookbook, where various results are given with very little rigorous justification. For mathematics students, on the other hand, linear algebra comes at a time when they are being introduced to abstract concepts and formal proofs as the foundations of mathematics. In the first case, very little of the fundamental concepts of LAMT is covered. For a second course in LAMT, there are a number of options. One option is to study the numerical aspects of LAMT, for example, in the book [11]. A totally different option, as in the popular book [14], views LAMT as a part of a basic abstract course in algebra.

This book is meant to be an introductory course in LAMT for beginning graduate students and an advanced (second) course in LAMT for undergraduate students. The first author has been teaching this course to both graduate and advanced undergraduate students, in the Department of Mathematics, Statistics and Computer Science, the University of Illinois at Chicago, for more than a decade. This experience has helped him to reconcile this dichotomy.

In this book, we use abstract notions and arguments to give the complete proof of the Jordan canonical form, and more generally, the rational canonical forms of square matrices over fields. Also, we provide the notion of tensor products of vector spaces and linear transformations. Matrices are treated in depth: stability of matrix iterations, the eigenvalue properties of linear transformations in inner product space, singular value decomposition, and min-max characterizations of Hermitian matrices and nonnegative irreducible matrices.

We now briefly outline the contents of this book. There are six chapters. The first chapter is a survey of basic notions. It deals with basic facts in LAMT, which may be skipped if the student is already familiar with them. The second chapter is a rigorous exposition of the Jordan canonical form over an algebraically closed field (which is usually the complex numbers in the engineering world) and a rational canonical form for linear operators and matrices. Again, the section dealing with cyclic subspaces and rational canonical forms can be skipped without losing consistency. Chapter 3 deals with applications of the Jordan canonical form of matrices with real and complex entries. First, we discuss the precise expression of $f(A)$, where A is a square matrix and f is a polynomial, in terms of the components of A. We then discuss the extension of this formula to functions f which are analytic in a neighborhood of the spectrum of A. The instructor may choose one particular application to teach from this chapter. The fourth chapter, the longest one, is devoted to properties of inner product spaces and special linear operators such as normal, Hermitian, and unitary. We study the min-max and max-min characterizations of the eigenvalues of Hermitian matrices, the singular value decomposition, and its minimal low-rank approximation properties. The fifth chapter deals with basic aspects of the Perron–Frobenius theory, which are not usually found in a typical linear algebra book. The last chapter is a brief introduction to the tensor product of vector spaces, the tensor product of linear operators, and their representation as the Kronecker product.

One of the main objectives of this book is to show the variety of topics and tools that modern linear algebra and matrix theory encompass. To facilitate the reading of this book we have included a good number of worked-out problems. These will help students in understanding the notions and results discussed in each section and prepare them for exams such as a graduate preliminary exam. We also provide a number of problems for instructors to assign to complement the material.

It may be hard to cover all of these topics in a one-semester graduate course. Since many sections of this book are independent, the instructor may choose appropriate sections as needed.

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Finally, we assume responsibility for any remaining typographical or mathematical errors, and we encourage readers to not only report to us any errors they find, but to also submit any other suggestions for improvements to future editions.

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