Preface

Numerical computation of highly oscillatory integrals came of age in the last 15 years. All three of us have been actively involved in this "highly oscillatory journey," which, in our view, has now reached a level of relative completeness, rendering it suitable for a broad review.

The box of tricks in the computation of highly oscillatory integrals—indeed, in the computation of a wider range of highly oscillatory phenomena, which includes, for example, highly oscillatory integral and differential equations—is fairly unusual to classically trained numerical analysts. This is because of its strong emphasis on asymptotic analysis in the oscillation parameter as a means to introduce numerical algorithms and, even more so, as a primary tool in *understanding* algorithms. The underlying mathematics is not very complicated, but it has different flavor to most texts in computational mathematics.

Theoretical interest, though, is neither the only nor the main motivation for this monograph. Highly oscillatory integrals feature in numerous applications, from fluid dynamics to mathematical physics, electronic engineering, acoustic and electromagnetic scattering, and so on, and the practical need to evaluate them is ubiquitous, often as part and parcel of a wider computational project. We thus hope that this monograph will be useful as well as illuminating, presenting a range of practical, highly efficient, and affordable algorithms.

We find the numerical analysis of highly oscillatory integrals a truly exciting chapter of computational mathematics, replete in unexpected and often counterintuitive ideas. Thus, we are so pleased to share it on these pages. While the theory has reached a stable stage which in our view makes it suitable for a review, nothing in mathematical research is truly final. We expect new ideas to come along (who knows, perhaps stimulated by this book) and add to the excitement. In particular, our review of Gaussian quadrature with a complex-valued measure in Chapter 6 represents work in a state of active exploration and clearly incomplete. We await further developments in this area.

All three of us have contributed to the state of the art in highly oscillatory quadrature, but we are far from alone: mathematical research is a collective enterprise. The story of highly oscillatory quadrature is no exception to this rule, and we are delighted to acclaim Andreas Asheim, Jing Gao, Arno Kuijlaars, Nele Lejon, David Levin, Syvert Nørsett, Sheehan Olver, Stefan Vandewalle, Haiyong Wang, and Shuhuang Xiang, whose work and insight have contributed so much to the state of the art and feature widely in this volume. And, of course, we must mention Louis Napoleon George Filon, whose ideas almost a century ago, serially forgotten or misinterpreted, fostered much of the modern theory and whose spirit (benignly, we hope) hovers over this volume.

Writing this book was a laborious process because all three of us are active academics, busy with the many preoccupations and challenges of academic existence, and

we have had to fit writing this book around our numerous other duties and projects. This means that deadlines have been serially missed, and we are truly grateful to SIAM's Executive Editor, Elizabeth Greenspan, for her infinite understanding and sorely tested, saintly patience.

Finally, we must pay tribute to our families for bearing with us during our work on this book. By now they must be used to our being busy, but in the last year they have had to get used to the three of us being *very* busy. They deserve our heartfelt thanks.

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