

# Index

- Abel–Weierstrass lemma, 413
- absolute condition number, 302
- addition
  - closure of, 5, 576
- adjoint, 121
- adjugate, 73, 76
- algebraic multiplicity, 143
- almost everywhere, 329, 332
  - convergence, 333
  - equal, 332
  - nonnegative, 340
- analytic function, 411, 414, 434
- angle preserving, 131
- arclength, 383
  - function, 384
  - parametrized by, 384
- argument principle, 445
- arithmetic-geometric mean inequality, 118
- Arnoldi
  - basis, 531
  - method, 547
- asymptotic behavior, 561
- automorphism
  - of a ring, 595
  - of a vector space, 38
- axiom of choice, 647
  
- Bézier curve, 54
- back substitution, 61
- Banach space, 211
- Banach-valued regulated integral
  - multivariable, 324
  - single-variable, 230
- barycentric
  - Lagrange interpolation, 610
  - weight, 610
  
- basic
  - columns, 65
  - variables, 63
- basis, 13
  - standard, 13, 51
- Bauer–Fike theorem, 560
- Beppo Levi theorem, 337
- Bernstein
  - operator, 82
  - polynomials, 54
- Bessel’s inequality, 97
- bijection, 38
- bijective, 38, 637
- bilinear, 89, 667
- block matrix
  - inverse, 666
  - multiplication, 665
- Bochner integral, 320
- Boolean ring, 575, 595
- boundary of a set, 192
- bounded
  - above, 645
  - functions, 113
  - linear transformation, 114, 186, 200, 235
  - linear transformation theorem, 213, 216
  - set, 197
- Brouwer fixed-point theorem, 225, 277
  
- calculus
  - fundamental theorem of, 403
- cancellation, 588
- canonical epimorphism, 41, 600
- Cantor
  - diagonal argument, 650
  - ternary set, 332
- cardinality, 648
- Cartesian product, 14, 19, 596, 630

- Cauchy
  - differentiation formula, 427
  - integral formula, 413, 424
  - sequence, 195, 196, 263
- Cauchy–Goursat theorem, 419, 421
- Cauchy–Riemann
  - equation, 410
  - equations, polar form, 451
  - theorem, 410
- Cauchy–Riemann equation, 410
- Cauchy–Schwarz inequality, 91
- Cayley–Hamilton theorem, 479
- chain, 20, 647
- chain rule
  - for holomorphic functions, 412
  - Fréchet derivative, 259
- change of basis, 53
- change of variables, 263, 351
- characteristic polynomial, 143
- Chinese remainder
  - problem, 602
  - theorem, 80, 601, 604
- classical adjoint, 76
- closed
  - $n$ -interval, 321
  - ball, 191
  - set, 190
- closure
  - of a set, 192
  - of operations, 5, 576
- codomain, 635
- cofactor, 73
  - expansion, 65, 75
- column space, 126
- commutative
  - ring, 575
- commutative diagram, 638
- compact, 179, 203
  - support, 122
- complementary
  - subspace, 16
- complete, 198
- completion, 361
- complex
  - conjugate, 654
  - numbers, 653
- composition, 36, 636
  - of continuous functions, 187
- computer aided design, 58
- condition, 302
- condition number
  - of a matrix, 305
  - absolute, of a function, 302
  - eigenvalue problem, 307
  - of a matrix, 307
  - relative, of a function, 303
- congruent
  - modulo  $n$ , 631
  - modulo an ideal, 598
- connected, 179, 224
  - component, 396
- conservative vector field, 388
- continuity
  - uniform, 199
- continuous
  - at a point, 185
  - function, 185, 188
  - linear extension theorem, 213, 216
  - Lipschitz, 186, 200, 235, 254, 278
  - pointwise, 187
  - uniformly, 199
- continuously differentiable, 266
- contour, 416
  - integral, 416, 417
  - simple closed, 419
- contraction
  - mapping, 278
  - mapping principle, 279
  - mapping principle, uniform, 282
  - mapping, uniform, 282
- convergence
  - absolute, 212
  - almost everywhere, 333
  - linear, 286
  - of sums, 212
  - pointwise, 210
  - quadratic, 286
  - uniform, 210, 413
  - uniform on compact subsets, 263
- convex, 261
- coordinate
  - change of, 299
- coordinates
  - change of, 48, 351
  - hyperbolic, 360
  - hyperspherical, 356

- in a basis, 47
  - polar, 352
  - spherical, 354
- coset, 23, 598
  - operations, 24, 599
- countable, 648
- cover
  - open, 203
- Cramer's rule, 77, 217
- cross product, 667
- CRT, *see* Chinese remainder theorem
- curl, 402
- curve
  - differentiable, 242
  - fitting, 129
  - piecewise-smooth, 382
  - positively oriented, 398
  - simple closed, 381
  - smooth, 382
  - smooth parametrized, 381
  - smooth, oriented, 382
  - smooth, unoriented, 382
- cutoff phenomenon, 564
- Daniell integral, 320, 328
- data compression, 168
- De Morgan's Laws, 630
- decay matrix, 564
- decomposition
  - LU, 62
  - polar, 165
  - QR, 103
  - singular value, 162
  - Wedderburn, 500
- dense, 190
- derivative, 242
  - directional, 244
  - higher, 266
  - linearity, 256
  - of a complex function, 408
  - of a parametrized curve, 242
  - second, 266
- determinant, 65
- De Moivre's formula, 655
- diagonal matrix, 162
- diagonalizable, 151
  - orthonormally, 157
- diffeomorphism, 349
- differentiable
  - complex function, 408
  - continuously, 253
  - function, 241, 246, 252
- dimension, 18
  - formula, 46
- direct sum, 14, 16
- Dirichlet function, 332
- divides, 585
- division property, 583
- domain
  - Euclidean, 583
  - of a function, 635
- dominated convergence theorem, 342
- dot product, 89
- Drazin inverse, 500, 501
- dual space, 248
- Eckart–Young, Schmidt, Mirsky
  - theorem, 167
- EEA, *see* extended Euclidean algorithm, 587
- eigenbasis, 151
- eigennilpotent, 483
- eigenprojection, 463, 475
- eigenspace, 140
  - generalized, 465, 468, 486
- eigenvalue, 140
  - semisimple, 492
  - simple, 307, 496
- eigenvector
  - generalized, 468
- elementary
  - matrix, 59
  - product, 68
- empty set, 627
- equivalence
  - modulo a subspace, 22
  - class, 22, 598, 632
  - modulo  $n$ , 21, 631, 633
  - modulo an ideal, 598
  - relation, 598, 631
- Euclidean
  - algorithm, 586
  - domain, 573, 583
  - extended algorithm, 587
- Euler's formula, 411, 415, 655
- extension by zero, 325

- extension theorem, 18, 29
- exterior of a simple closed curve, 397
- extreme value theorem, 205
- Fatou's lemma, 340
- field, 659
- finite intersection property, 206
- first isomorphism theorem
  - for rings, 600
  - for vector spaces, 42
- fixed point, 278
- formula
  - Euler, 411
- formal power series, 576
- Fourier transform, discrete, 622
- Fréchet derivative, 246, 252
  - chain rule, 259
  - higher-order, 266
  - product rule, 257
  - real finite-dimensional case, 246
- Fredholm
  - alternative, 136
  - integral transform, 281
- free variables, 63
- Frobenius
  - inner product, 90, 136
- Fubini's theorem, 320, 344
  - for step functions, 372
- function, 635
  - analytic, 411, 413
  - entire, 408
  - holomorphic, 408
  - integrable (Daniell–Lebesgue), 330
  - regulated integrable, 324, 375
  - Riemann integrable, 320, 325, 340
  - smooth, 266
- fundamental
  - subspaces theorem, 124
  - theorem of algebra, 430, 591
  - theorem of arithmetic, 590
  - theorem of calculus, 262
  - theorem of calculus for line
    - integrals, 388
- fundamental theorem
  - of calculus, 403
- Gamma function, 359
- Garner's formula, 606
- Gauss' mean value theorem, 426
- Gauss–Seidel
  - convergence, 524
  - method, 522
- gcd, 586
- generalized
  - eigenspace, 465, 468, 486
  - Heine–Borel theorem, 208
  - inverse, *see* pseudoinverse
  - Leibniz integral rule, 348
  - minimal residual method (GMRES), 530
  - step function, 379
- geometric multiplicity, 141
- Givens rotation, 545
- GMRES, 530, 533
- Google, 498
- Gram–Schmidt, 99
  - modified, 101
- graph, 635
- greatest
  - common divisor, 586
  - element, 646
  - lower bound, 645
- Green's theorem, 381, 399
- Hölder's inequality, 118
- Heine–Borel theorem
  - generalized, 208
  - on  $\mathbb{R}^n$ , 204
- Hermitian conjugate, 84, 664
- Hessian matrix, 266
- Hilbert's identity, 471
- Hilbert–Schmidt
  - inner product, 90
- holomorphic
  - function, 408, 434
  - open mapping theorem, 445, 450
- homeomorphism, 222
- Householder, 105
- Householder transformation, 105, 106
- hyperbolic coordinates, 360
- hyperplane, 321
- hyperspherical coordinates, 356
- ideal, 578
  - generating set, 581
- identity map, 39

- ill conditioned, 303
- image, 594, 635
- implicit function theorem, 294
- index of a matrix, 466
- index set, 639
- indicator function, 228, 323
- induced
  - metric, 208
  - norm on linear transformations, 114
  - norm, from an inner product, 111
- induction, 644
- infimum, 645
- inherited metric, 208
- injective, 637
- inner product, 88
  - Frobenius, 90, 136
  - Hilbert–Schmidt, 90
  - positivity of, 91
  - space, 89
  - standard, 89
- integers, 628
- integrable
  - function, Daniell–Lebesgue, 330
  - function, regulated, 324
  - function, Riemann, 320, 325, 340
  - function, regulated, 375
  - on an unbounded domain, 337
- integral
  - Bochner, 320
  - contour, 417
  - Daniell, 320, 328
  - iterated, 345
  - Lebesgue, 320, 328
  - mean value theorem, 262
- interior
  - of a set, 182
  - of a simple closed curve, 397
- interior point, 182
- intermediate value theorem, 225
- intersection of sets, 630
- invariant subspace, 147, 462, 468
- inverse function theorem, 298
- invertible, 38
  - element, 578
  - left, 642
  - right, 642
- involution, 172
- irreducible
  - element, of a ring, 588
  - matrix, 497
- isolated point, 189
- isometric embedding, 362
- isometry, 116
- isomorphic rings, 596
- isomorphism
  - of rings, 595, 596
  - of vector spaces, 36, 38
- iterated integral, 345
- iterative
  - numerical methods, 519
  - solvers, 521
- iterative methods, 520
- Jacobi method, 522
  - convergence, 524
- Jacobian
  - determinant, 297
  - matrix, 249
- Jordan
  - canonical form, 140, 506
  - curve theorem, 396
- Jordan normal form, 480
- Kantorovich, 315
- kernel, 35, 594
- Kreiss constant, 562
- Kreiss matrix theorem, 563
- Kronecker delta, 95
- Krylov
  - basis, 527
  - methods, 526
  - solution, 529
  - subspace, 506, 527
- Lagrange
  - basis, 603
  - decomposition, 602
  - interpolant, 603
  - interpolant, barycentric form, 610
  - interpolation, 610
  - interpolation, barycentric, 610
- Lagrange–Hermite interpolation, 616, 617
- Lanczos method, 548
- Laplace equation, 145

- Laurent
  - expansion, 436
  - polynomial, 576
  - series, 433, 436
  - series, principal part, 438
- law of cosines, 92
- leading entry, 61
- least
  - element, 647
  - upper bound, 645
  - upper bound property, 646
- least squares, 127
- Lebesgue
  - integrable, 367
  - integral, 320, 328
  - number, 206
- left
  - eigenvector, 154
  - invertible, 642
  - pointing normal vector, 398
- Legendre polynomial, 101, 453
- Leibniz integral rule
  - generalized, 348
- Leibniz's integral rule, 346
- Leontief, Wassily, 520
- level set, 191, 293, 635
- limit
  - inferior, 340
  - of a function, 188
  - of a sequence, 193
  - point, 189
  - pointwise, 210
  - superior, 340
- line fitting, 129
- line integral, 386
  - of scalar field, 386
  - of vector field, 387
- linear
  - approximation, 242
  - combination, 10
  - convergence, 286
  - dependence, 12
  - extension, 230
  - functional, 121
  - homogeneous system, 40
  - independence, 12
  - operator, 33
  - ordering, 644
  - system, 32, 58, 105, 519, 520, 527, 530
    - homogeneous, 35, 40
    - overdetermined, 127
  - transformation, 31–33
    - bounded, 114
    - composition of, 51
    - matrix representation, 49
- Liouville's theorem, 430
- Lipschitz
  - continuous, 186, 200, 235, 254, 278
  - locally at a point, 254
- lower bound, 645
- l.u.b. property, *see* least upper bound property
- LU decomposition, 62
- manifold, 381
- map, 635
- Markov chain, 564
- matrix, 663
  - $\infty$ -norm, 116
  - 1-norm, 116
  - augmented, 60
  - block, 665
  - decay, 564
  - diagonalizable, 151
  - diagonally dominant, 524
  - Hermitian conjugate, 664
  - index of, 466
  - invertible, 665
  - irreducible, 497
  - minor, 73
  - nilpotent, 171
  - nonnegative, 494
  - nonsingular, 52, 665
  - normal, 158
  - orthogonal, 98
  - orthonormal, 98, 132
  - positive, 494
  - primitive, 497
  - product, 663
  - row equivalent, 61
  - semisimple, 151
  - similar, 53
  - simple, 151
  - sparse, 519

- transition, 53
  - transpose, 664
  - tridiagonal, 145
  - unitary, *see* matrix, orthonormal
- maximal element, 646
- maximum modulus principle, 431
- mean value theorem, 260
  - Gauss's, 426
  - integral, 262
- measurable
  - function, 333
  - set, 331, 333
- measure, 323
  - zero, 331
  - zero, alternative definition, 364
- meromorphic, 439
- method of successive approximations, 280
- metric, 179, 180
  - Cartesian product, 181
  - discrete, 181
  - Euclidean, 180
  - French railway, 233
  - induced, 208
  - inherited, 208
  - normalized, 181
  - space, 180
- minimal polynomial, 526
- minimum modulus principle, 453
- Minkowski's inequality, 119
- Mirsky, Schmidt, Eckart–Young theorem, 167
- modulus
  - componentwise, of a matrix, 513
  - of a complex number, 654
- monic polynomial, 143, 586
- monotone
  - convergence theorem, 336
  - decreasing, 336
  - increasing, 201, 336
- Moore–Penrose pseudoinverse, 126, 166
- multiplication
  - closure of, 576
- multiplicity
  - of a zero, 435, 446
  - of an eigenvalue
    - algebraic, 143, 156, 469
    - geometric, 141, 156, 467
- natural numbers, 628
- neighborhood, 182
- Neumann series, 215
- Newton
  - decomposition, 604
  - interpolation, 611
- Newton's method
  - scalar version, 287
  - vector version, 291
- Newton–Kantorovich theorem, 292, 315
- noise canceling, 10
- nonsingular matrix, 665
- norm, 111
  - $L^p$ -, 113
  - $p$ -, 112
  - Euclidean, 111
  - Frobenius, 113, 168
  - induced
    - from an inner product, 111
    - on linear transformations, 114
  - Manhattan, 111
  - matrix, 115
  - operator, 114
  - sup, 113
  - taxicab, 111
- normal
  - distribution, 359
  - equation, 128
  - matrix, 158
- normed linear space, 111
- null space, 35
- nullity, 43
- numerical
  - linear algebra, 62
- oblique projection, 460
- one-to-one, 637
- onto, 637
- open
  - ball, 182
  - cover, 203
  - set, 182
- order
  - of a nilpotent operator, 485
  - of a pole of a function, 439
  - of a zero of a function, 435
- orderings, 643

- orthogonal, 91
  - complement, 105, 123
  - projection, 93, 96
- orthonormal, 87
  - matrix, 98, 132
  - set, 95
  - transformation, 97
- orthonormally
  - diagonalizable, 157, 158
  - similar, 155
- outer product expansion,
  - 164, 174
- overdetermined, 127
  
- PageRank algorithm, 498
- parallelogram identity, 131
- parametrized
  - contour, 416
  - curve
    - equivalent, 382
    - smooth, 381
  - manifold, 389
    - equivalent, 389
    - measure of, 394
    - oriented, 389
    - tangent space, 390
    - unoriented, 390
  - surface, 389
- parametrized manifold, 381
- partial
  - derivative, 245
    - $k$ th-order, 266
    - in a Banach space, 255
  - ordering, 646
  - sums, 212
- partially ordered set, 646
- partition, 22, 598, 633
- path, 416
  - connected, 226
  - independent, 420
- periodic function, 236
- permutation, 66
  - even, 67
  - inversion, 67
  - odd, 67
  - sign, 67
  - transposition, 67
  
- Perron
  - root, 496
  - theorem, 496
- Perron–Frobenius
  - eigenvalue, 496
  - theorem, 497
- piecewise smooth, 382, 416
- pigeonhole principle, 652
- polar decomposition, 165
- polarization identity, 131
- pole of a function, 439
- polygonal path, 421
- polynomial
  - monic, 143, 586
  - ring, 575
- poset, *see* partially ordered set
- positive
  - definite, 159
  - semidefinite, 159
- potential function, 388
- power
  - method, 492
  - set, 628
- preimage, 186, 635
- preimage of a function, 187
- prime, 588
- primitive
  - matrix, 497
- primitive root of unity, 657
- principal ideal domain
  - Euclidean domain is a, 585
- projection, 136, 460, 639
  - canonical, 597
  - complementary, 460
  - map, 33
  - spectral, 475
- pseudoeigenvalue, 554
- pseudoeigenvector, 555
- pseudoinverse
  - Drazin, 500, 501
  - Moore–Penrose, 166
- pseudospectral
  - radius, 561
- pseudospectrum, 554
- Pythagorean
  - law, 92
  - theorem, 97



## QR

- decomposition, 102, 103
- decomposition, reduced, 103
- iteration, 538

quadratic convergence, 286

quasi-Newton method, 290

quotient

- of a ring by an ideal, 598
- of a vector space by a subspace, 23

$R$ -linear combination, 581

radius

- pseudospectral, 561

radius of convergence, 414

range, 35

rank, 43

rank-nullity theorem, 43

Rayleigh quotient, 174

reduced row echelon form, 61

REF, *see* row echelon form

reflection, 172

regulated

- integrable functions, 324
- integral
  - multivariable, Banach valued, 324
  - single variable, Banach valued, 228

relation, 631

relative condition number, 303

relatively prime, 586

reparametrization, 382

replacement theorem, 17

residual vector, 93

residue

- of a function, 440
- theorem, 442

resolvent, 470

- of an operator, 470
- set, 141, 470

reverse Fatou lemma, 358

Riemann integral, 325

Riemann's theorem, 426

Riesz representation theorem, 120

right

- eigenvector, 154
- invertible, 642

ring, 574

- commutative, 575, 576
- homomorphism, 592
- of polynomials in a matrix, 576

Ritz eigenvalues, 547

root

- of unity, 657
- simple, 304

rotation map, 34

Rouché's theorem, 448

row

- echelon form, 61
- echelon form, reduced, 61
- operations, 58
- reduction, 58

RREF, *see* row echelon form, reduced

Russell's paradox, 629

scalar, 4

Schmidt, Mirsky, Eckart–Young theorem, 167, 168

Schur

- complements, 666
- form, 140, 156
- lemma, 155

second

- barycentric form, 611
- derivative, 266
- isomorphism theorem, 45

segment, 648

self-adjoint, 155

seminorm, 111, 361

semisimple

- eigenvalue, 492
- matrix, 151
- spectral mapping theorem, 153

separated

- spectrally, 540

sequence, 193

- Cauchy, 195, 263
- convergent, 193

uniformly convergent, 210

sequentially compact, 206

series, 212

sesquilinear, 89

set, 627

- complement, 630
- difference, 630

- intersection, 630
  - of measure zero, 331
  - partition, 633
  - union, 629
- Sherman–Morrison–Woodbury lemma, 666
- shifting
  - QR iteration, 550
- sign of a complex number, 656
- similar matrix, 53
- simple
  - closed curve, 381
  - eigenvalue, 307, 496
  - matrix, 151
  - pole, 439
  - region, 398
  - root, 304
- simply connected, 397
- singular
  - matrix, 52
  - value decomposition, 162
    - compact form, 164
  - values, 162
- singularity
  - essential, 439
  - isolated, 438
  - removable, 439
- skew-Hermitian, 171
- small gain theorem, 171
- smooth, 266
- SOR, *see* successive overrelaxation
- span, 10
- spectral
  - condition number, 560
  - decomposition theorem, 487
  - mapping theorem, 489
  - radius, 474, 478
  - resolution formula, 478
  - theorem
    - Hermitian matrices, 157
    - normal matrices, 158
- spectrally separated, 540
- spectrum, 141, 470
- spherical coordinates, 354
- Spijker’s lemma, 566
- Stein map, 34
- step function, 228, 323
  - generalized, 379
  - integral of, 229, 324
- Stokes’ theorem, 402
- subcover
  - finite, 203
- subdivision
  - generalized, 321
  - of intervals, 321
  - one-dimensional, 228
- submatrix, 73
- submultiplicative property, 115
- subsequence, 197
- subset, 628
  - proper, 628
- subspace, 7
  - complementary, 14, 16
  - invariant, 147
  - proper, 9
  - trivial, 9
- substitution formula, 349
- successive approximation, 280
- successive overrelaxation
  - convergence, 525
- superposition, 9
- support, 635
- supremum, 645
- surface
  - parametrized, 389
- surjective, 637
- SVD, *see* singular value decomposition
- Sylvester map, 34
- symmetric group, 66
- tangent
  - to a curve, 382
  - unit, 382
- tangent vector, 243
- Taylor
  - formula, 268, 269
  - remainder, 271
  - series for holomorphic functions, 433
- topological
  - properties, 179, 222, 223
  - spaces, 186

- topologically equivalent, 219, 220
  - norms, 134, 221
- topology, 179, 186
  - induced, 219
- total
  - derivative, 246
  - ordering, 643
- totally bounded, 206, 208
- transient behavior, 561
- transition matrix, 48, 53
- transpose, 664
- transposition matrix, 59
- tridiagonal matrix, 145
  
- unbounded linear transformation, 235
- uncountable, 648
- uniform
  - continuity, 195, 199
  - contraction mapping, 282
  - contraction mapping principle, 282
  - convergence, 210, 413
  - convergence on compact subsets, 263
  - properties, 223
- union of sets, 629
- unit, 578
  - normal, 391, 392
  - tangent, 382
  - vector, 91
- unitarily similar, 155
- unitary matrix, *see* orthonormal matrix
- unity
  - ring, 577
  
- universal property of the product, 639
- upper bound, 645
  
- valuation, 583
- variables
  - change of, 351
- vector, 4
  - field, 387
    - conservative, 388
  - length of, 91
  - space, 4
  - unit, 91
- vectors
  - angle between, 92
- volume, 331, 351
  - $n$ -dimensional, 323
  - of  $n$ -ball, 360
  - of a parallelepiped, 394
  
- Wedderburn decomposition, 500
- well conditioned, 303
- well-defined functions, 640
- well-ordering axiom, 644
- well-ordering principle, 647
- winding number, 441
- WOA, *see* well-ordering axiom
- Wronskian, 85
  
- Young's inequality, 118
  
- zero
  - and pole counting formula, 446
  - divisors, 583
  - of order  $n$ , 435
- Zorn's lemma, 20, 647