

Index

- ACF, *see* autocorrelation function
- adaptive Metropolis, 91
 - for marginal sampling, 93
- adaptive proposal, 91
- adjoint method, 108–109
- AM, *see* adaptive Metropolis
- anisotropic
 - diffusion, 60
 - increments, 60
 - prior, 60, 63
 - total variation, 66
- array, 35
- autocorrelation function, 87

- Bayes' law, 54, 61, 82, 106, 119
- BCCB, *see* block circulant with circulant blocks
- bias, 10, 19
- biharmonic operator, 70, 71
- block circulant with circulant blocks, 37, 39, 57, 63
 - diagonalize, 39
- block Toeplitz with Toeplitz blocks, 37
- boundary conditions, 3
 - data driven, 28, 40
 - Neumann, 28, 42
 - periodic, 26, 37
 - zero, 3, 34
- BTTB, *see* block Toeplitz with Toeplitz blocks
- burn-in, 84

- cdf, *see* cumulative distribution function
- Cholesky factorization, 78–80, 93
- circulant matrix, 26
 - diagonalize, 27, 30
- computed tomography, 42
- conditional autoregression, 54

- conjugate gradient iteration
 - preconditioned, 45
 - sampling from the posterior, 81
 - within hierarchical Gibbs, 97
- conjugate prior/hyper-prior, 83
- convolution, 2, 26, 33
- costate equation, 109
- covariance
 - empirical or sample, 91
- covariance matrix, 6
- cross validation, *see* generalized cross validation
- CT, *see* computed tomography
- cumulative distribution function, 117

- data-driven boundary conditions, 28, 40
- data models
 - linear, additive Gaussian, 1
 - nonlinear, additive Gaussian, 105
- deblurring, *see* deconvolution
- deconvolution, 2, 33
- degeneracy in hierarchical Gibbs, 88
- detailed balance, 90
- DFT, *see* discrete Fourier transform
- diag operator, 16
- diagonalize
 - block circulant with circulant blocks, 39
 - circulant, 27
- difference matrix, 5
 - Neumann boundary condition, 58
 - periodic boundary condition, 63
- discrepancy principle, 22–23, 37, 46

- discrete Fourier transform, 27, 39
- discrete Picard condition, 13
- discretization
 - computed tomography, 43
 - convolution, 3, 26, 34
 - kernel reconstruction, 5
 - negative-Laplacian, 72–73
- DP, *see* discrepancy principle

- edge-preserving prior, 63
- eigenvalue decomposition, 78
- empirical covariance, 91
- empirical quantile, 80
- essential sample size, 87
- expected value, 6

- fast Fourier transform, *see* discrete Fourier transform
- filter factors, 16
 - Landweber iterations, 18
 - Tikhonov, 17
 - TSVD, *see* truncated singular value decomposition
- filtered SVD, 16
- finite difference discretization of negative-Laplacian, 72–73
- full conditionals, 54, 83, 119
- full posterior density function, 83, 119

- gamma distribution, 84
- Gauss–Newton
 - Hessian, 107, 108
 - method, 107
- Gaussian Markov random field, 54–61
 - anisotropic, 59–61, 63
 - edge-preserving, 63
 - higher-order, 70
 - iid increments, 58

- intrinsic, 58
- Neumann boundary condition, 58
- periodic boundary condition, 57
- sampling from, 78
- zero boundary condition, 56
- Gaussian posterior density
 - sampling from, 79–81
- Gaussian random vector, 6
 - nonlinear transformation of, 110
- GCV, *see* generalized cross validation
- generalized cross validation, 21–22, 40, 46, 62
- generalized inverse, 8
- Geweke test, 84
- Gibbs sampler
 - hierarchical, *see* hierarchical Gibbs sampler
 - partially collapsed, 95
 - two-stage, 83
- GMRF, *see* Gaussian Markov random field
- gradient, 107, 108
- gradient scan Gibbs, 98

- Hessian, 68, 107, 108
- hierarchical Gibbs sampler, 83
 - degeneracy, 88
 - with embedded PCG, 97
 - with embedded RTO-MH, 118
- hierarchical modeling, 81, 118
- hyper-parameters, 120
- hyper-prior probability density, 82

- IACT, *see* integrated autocorrelation time
- iid, *see* independently and identically distributed, 65
- ill-posedness, 10
 - inequality, 7, 10, 11, 106
- increments, 59
 - anisotropic, 60
 - first order, 58
 - iid Gaussian, 58
 - Laplace, 65
 - second order, 70
- independence proposal, 91
- independently and identically distributed, 1
- infinite-dimensional limit
 - hierarchical Gibbs degeneracy, 88
 - well-defined posterior density, 68
- integrated autocorrelation time, 87
- invariant density, 90
- inverse problems, 2
 - test cases, *see* test cases
- iterative regularization, 18

- Jacobian, 68, 107, 108, 110, 112, 115

- Kaczmarz’s method, 50–51
- kernel, 2, 33
 - Gaussian, 3, 34
 - separable, 34
- kernel reconstruction test case, 4
 - with positivity constraints, 116
- Kronecker product, 35

- L-curve, 23–25, 46
- lagged-diffusivity fixed point iteration, 67
- Landweber iteration, 18
- Laplace prior, 65
 - transformation, 117
- Laplacian, negative, *see* negative-Laplacian
- least squares estimator
 - linear, 6
 - nonlinear, 105
 - normal equations, 6
 - statistical properties, 7, 8
 - SVD form, 8
- Levenburg–Marquardt, 106–108
- lexicographical ordering, 35, 63
- likelihood function, 6, 54, 82, 105
- LM, *see* Levenburg–Marquardt
- log-normal prior, 116

- MAP estimator, *see* maximum a posteriori estimator
- marginal density, 84, 89
- Markov chain diagnostics, *see* MCMC chain diagnostics
- Markov chain Monte Carlo method, 77
- matrix
 - block circulant with circulant blocks, 37, 39, 57, 63
 - block Toeplitz with Toeplitz blocks, 37
- circulant, 26
- covariance, 6
- difference, 5, 58, 63
- precision, 55
- symmetric positive definite, 56, 78
- Toeplitz, 3, 34
- maximum a posteriori estimator, 54, 61, 106
- maximum likelihood estimator, 6, 105
- MCE, *see* Monte Carlo error
- MCMC chain diagnostics
 - autocorrelation function, 87
 - burn-in, 84
 - equilibrium, 85
 - essential sample size, 87
 - Geweke test, 84
 - integrated autocorrelation time, 87
- MCMC method, *see* Markov chain Monte Carlo method
- mean, 6
- mean squared error, 9, 19
- Metropolis–Hastings, 90–91
 - within hierarchical Gibbs, 118
 - with RTO proposal, 111
- MH, *see* Metropolis–Hastings
- midpoint quadrature, 3, 5, 12, 26, 33, 38
- Monte Carlo error, 86
- MSE, *see* mean squared error

- negative-Laplacian
 - finite difference discretization, 72–73
 - Neumann boundary conditions, 59
 - periodic boundary conditions, 57
 - zero boundary conditions, 56
- neighborhood system, 55
- Neumann boundary conditions
 - deconvolution, 28
 - GMRF, 58
 - negative-Laplacian, 59
- Newton’s method, 67, 106
- nonlinear least squares, 105, 106
- normal equations, 6

- one-block algorithm, 93
- optimization problems
 - MAP estimator, linear, 67

- MAP estimator, nonlinear, 106
- randomize-then-optimize, 111
- outer product form
 - pseudoinverse, 8
 - SVD, 8
- partially collapsed Gibbs, 95
- PCG, *see* preconditioned conjugate gradient
- periodic boundary conditions
 - convolution, 26, 37
 - GMRF, 57
 - negative-Laplacian, 57
- point spread function, *see* kernel
- Poisson equation inverse problem, 113, 120
- positivity constraints, 116
- posterior density function, 54, 106
 - hierarchical, full, 83, 119
 - sampling from Gaussian, 79–81
- precision
 - matrix, 55
 - parameter, 82
- preconditioned conjugate gradient, *see* conjugate gradient iteration
- preconditioner, 45, 65
- prior
 - improper, 58
- prior probability density
 - edge-preserving, 63
 - Gaussian Markov random field, 54–61, 82, 106
 - iid Gaussian, 54
 - Laplace (total variation), 65
 - log-normal, 116
 - smoothness, 55
 - uniform, 92
 - Whittle–Matérn class, 71
- proposal
 - adaptive, 91
 - independence, 91, 111
 - symmetric, 91
- proposal density, 90
- pseudoinverse, 8
- QR-factorization, 110
- quasi-Newton method, 68
- Radon transform, 43
 - discrete, 43
- randomize-then-optimize, 109–112
- Metropolis–Hastings, 111
- MH within hierarchical Gibbs, 120
 - optimization problem, 111
 - probability density, 110
- randomized trace estimation, 46
- regression, 1
 - linear, 1
 - principle component, 16
 - ridge, 17
- regularization parameter, 16
- regularization parameter selection methods
 - discrepancy principle, 22–23
 - generalized cross validation, 21–22
 - L-curve, 23–25
 - unbiased predictive risk estimator, 20–21
- regularized solution
 - edge-preserving, 63
 - filtered SVD, 16
 - Landweber iteration, 18
 - statistical properties, 19
 - total variation, 66
 - TSVD, 16
- residual, 107
- ridge regression, 17
- RTO, *see* randomize-then-optimize
- sampling from
 - full posterior density, 83, 95
 - Gaussian Markov random field, 78
 - Gaussian posterior density, 79–81
 - marginal density, 93
 - posterior with Laplace prior, 117
 - posterior with positivity constraints, 116
- semiconvergence, 18
- separable kernel, 34
- signal-to-noise ratio, 4
- singular value decomposition, 7
 - filtered, 16
 - properties, 13
 - truncated, 16
- singular values, 8
- singular vectors, 8
- SNR, *see* signal-to-noise ratio
- SPD, *see* symmetric positive definite
- spectral filtering, 16
- statistical properties
 - least squares solution, 8
 - regularized solutions, 19
- stochastic
 - linear system, 81
 - optimization problem, 111
- SVD, *see* singular value decomposition
- symmetric proposal, 91
- test cases
 - computed tomography, 42
 - deconvolution, 2, 33
 - kernel reconstruction, 5
 - Laplace prior, 117
 - Poisson equation, 113
 - positivity constraints, 116
 - two-parameter, 11, 91, 112, 121
- Tikhonov
 - pseudonormal equations, 17, 42, 44
 - regularization, 17
 - regularized solution, 17, 27, 37, 40, 54
- Toeplitz matrix, 3, 12, 25, 34
- total variance, 9, 19
- total variation, 66
- trace class operator, 69
- trace lemma, 20
- transformation
 - of a Gaussian random vector, 110
 - of the Laplace prior, 117
- transition kernel, 90
- truncated singular value decomposition, 16–17
- unbiased predictive risk estimator, 20–21, 37, 46
- underdetermined, 3, 28
- uniform
 - distribution, 91
 - prior, 92
- UPRE, *see* unbiased predictive risk estimator
- vec operator, 35
- wavelet transform, Haar, 118
- Whittle–Matérn class of priors, 71
- zero boundary conditions
 - convolution, 3, 34
 - GMRF, 56
 - negative-Laplacian, 56