# Preface

Many physical, chemical, biomedical, and technical processes can be described by means of partial differential equations (PDEs) or dynamical systems. If the underlying processes exhibit nonlinear dynamics, analysis and prediction of the complex behavior is often only possible by solving the (partial) differential equations numerically. For this reason, the design of efficient numerical schemes is a central research challenge. In spite of increasing computational capacity, many problems are of such high complexity that they are still only solvable with severe simplifications. In recent years, large-scale problems—often involving multiphysics, multiscale, or stochastic behavior—have become a particular focus of applied mathematical and engineering research. A numerical treatment of such problems is usually very time-consuming and thus requires the development of efficient discretization schemes that are often realized in large parallel computing environments. In addition, these problems often need to be solved repeatedly for many varying parameters, introducing a curse of dimensionality when the solution is also viewed as a function of these parameters. With this book we aim to introduce recent developments on complexity reduction of such problems, both from a theoretical and an algorithmic perspective.

This book is based partially on keynote lectures from the workshop "Model Reduction and Approximation for Complex Systems," held at the Centre International de Rencontres Mathématiques, June 10–14, 2013, in Luminy, Marseille, France. This workshop brought together some of the world's leading experts from mathematics and engineering sciences who are concerned with modeling, approximation, and model reduction of complex (parametrized) systems. In particular, the focus was on further developing and analyzing approaches for practically relevant problems that are modeled by PDEs or dynamical systems.

The workshop addressed complexity reduction of such problems in settings that include design, control, optimization, inverse problems, uncertainty analysis, and statistical sampling. In recent years, there has been a tremendous effort to develop efficient approaches to deal with such problems in various mathematical communities. In particular, relevant research areas are high-dimensional and sparse approximation, system-theoretic model order reduction (MOR) for dynamical systems, proper orthogonal decomposition (POD), reduced basis (RB) methods for parametrized PDEs, numerical multiscale methods, polynomial chaos approximations, stochastic finite elements (FEs), approximation by slow manifolds, multiresolution methods, and hierarchical dimension reduction techniques.

The major goal of the workshop was to bring together leading mathematicians and engineers from these research areas to foster collaboration and to stimulate an exchange of ideas among these communities, with the aim to act as a catalyst for new and innovative ideas in this challenging field of research.

The main purpose of the book resulting from the workshop is to extend the keynote lectures of the workshop to tutorials accessible to developers and users of mathematical methods for model reduction and approximation of complex systems. In addition to the keynote lecturers of the workshop, we also invited other experts to contribute chapters on methods not represented in the keynotes. The book thus contains tutorial-style introductions to several promising emerging fields in model reduction and approximation. It focuses in particular on sampling-based methods (Part I), such as the RB method and POD, approximation of high-dimensional problems by low-rank tensor techniques (Part II), and system-theoretic methods (Part III), such as balanced truncation (BT), interpolatory methods, and the Loewner framework.

Both application-driven aspects and fundamental points of view from approximation theory and information-based complexity are discussed. This reveals the great success of the proposed techniques for certain classes of applications but at the same time also shows their limitations. Real-life problems often pose major challenges that are currently covered by neither the mathematical theory nor the presented methods and thus constitute a driving force for future research.

We believe that the chapters collected in this book exhibit high tutorial value, so that in combination they can serve as the basis of graduate-level courses on the subject. Such courses could be valuable in any master's-level program ranging from applied and engineering mathematics to computational science and engineering. The tutorials could also be used as course material for integrated PhD programs or summer schools.

The second purpose of the book is to serve as a reference guide for examples and methods available to date, in particular for parametric MOR. It compares some of the methods for parametric model reduction using examples collected at the MOR Wiki.[1] Thus, it provides a first guide to the choice of suitable algorithms for model and complexity reduction of dynamical systems. This is merely a starting point, and further developments will be made available through the MOR Wiki.

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[1] See `http://morwiki.mpi-magdeburg.mpg.de/`.