Introduction

This volume contains lectures presented at the Park City summer school on "Mathematics and Materials" in July 2014. The central theme is a description of material behavior that is rooted in statistical mechanics. While many presentations of mathematical problems in materials science begin with continuum mechanics, these lectures present an alternate view. A rich variety of material properties is shown to emerge from the interplay between geometry and statistical mechanics.

The school included approximately eighty graduate students and forty researchers from many areas of mathematics and the sciences. This interdisciplinary spirit is reflected in a diverse set of perspectives on the order-disorder transition in many geometric models of materials, including nonlinear elasticity, sphere packings, granular materials, liquid crystals, and the emerging field of synthetic self-assembly.

The lecturers for the school, and the topics of their lectures, were as follows:

(1) Michael Brenner, School of Engineering and Applied Sciences, Harvard University: Ideas about self-assembly.
(2) Henry Cohn, Microsoft Research: Packing, coding and ground states.
(3) Veit Elser, Department of Physics, Cornell University: Three lectures on statistical mechanics.
(4) Daan Frenkel, Department of Chemistry, University of Cambridge: Entropy, probability and packing.
(5) Richard D. James, Department of Aerospace Engineering and Mechanics, University of Minnesota: Phase transformations, hysteresis and energy conversion –the role of geometry in the discovery of materials.
(6) Robert V. Kohn, Courant Institute, New York University: Wrinkling of thin elastic sheets.
(7) Roman Kotecký, Mathematics Institute, University of Warwick: Statistical mechanics and nonlinear elasticity.
(8) Peter Palffy-Muhoray, Department of Chemical Physics, Kent State University: The effects of particle shape in orientationally ordered soft materials.

In addition, L. Mahadevan (Harvard University) and Felix Otto (MPI, Leipzig) were in residence for the program as Clay Senior Scholars, and gave well-received public lectures.

All the lectures in this volume contain unique pedagogical introductions to a variety of topics of current interest. Several lectures touch on the interplay between discrete geometry (especially packing) and statistical mechanics. These problems have an immediate mathematical appeal and are of increasing importance in applications, but are not as widely known as they should be to mathematicians with

an interest in materials science. Both Elser and Frenkel present elegant introductions to statistical mechanics from the physicist's perspective, with an emphasis on the interplay between entropy and packings. This theme is repeated in Cohn's lectures, which reveal the role of unexpected mathematical tools in simply stated problems about symmetric ground states. Similarly, Brenner uses discrete geometry and statistical mechanics to model exciting new experiments on synthetic self-assembly. Palffy-Muhoray uses statistical mechanics to derive several models for liquid crystals, exploring again the interplay between shape and statistical mechanics. Kotecký's lecture contains a mathematical introduction to statistical mechanics, with a focus on the foundations of nonlinear elasticity. The volume also includes an account of recent work on correctors in stochastic homogenization by Otto and his co-workers. Regrettably, this volume does not contain the texts of excellent lectures by James on solid-solid phase transitions, and by Kohn on the elasticity of thin sheets.

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